PART – A

Answer ALL questions : **(10 x 2 = 20)**

1. Evaluate $\int \frac{1}{x\,(\log x)^n}\,dx$.
2. Prove that $\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = \frac{\pi}{4}$.
3. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.
4. Solve $ydx - xdy + 3x^2y^2e^{x^3}dx = 0$.
5. Test the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots \infty$.
6. State the Limit comparison test .
7. Write the expansion of $(1+x)^{-2}$.
8. Find the coefficient of x^n in the expansion of e^{a+bx} .
9. What are the intercepts made by the plane $2x + 3y + 5z + 7 = 0$ on the coordinate planes ?
10. Find the equation of the line through the point $(3,\ 2, -1)$ and perpendicular to the plane $5x - 4y + 7z - 1 = 0$.

PART – B

Answer any FIVE questions : **(5 x 8 = 40)**

11. Evaluate $\int \frac{xe^x}{(x+1)^2}\,dx$.
12. Solve $(D^2 - 3D + 2)y = \sin 3x$.
13. Solve $3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$.
14. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin\frac{1}{n}$.
15. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$.
16. If x and y are small, show that $\frac{(1+y)^x}{(1+x)^y} = 1 + \frac{1}{2}xy(x-y)$.
17. Show that $\frac{1}{1.\,2.\,3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \ldots\ldots \infty = \log 2 - \frac{1}{2}$.
18. Find the equation of the plane passing through the points (9, 3, 6) and (2, 2, 1) and perpendicular to the plane $2x + 6y + 6z - 9 = 0$.

PART-C

Answer any TWOquestions : **(2 x 20 = 40)**

19. (a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, prove that $I_n + I_{n-2} = \frac{1}{n-1}$ and hence evaluate I_5.

(b) Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x$. (10 +10)

20. (a) Solve $(D^2 + 2D + 5)y = xe^x$.

(b) Test the convergence of the series $\frac{1}{3} + \frac{2}{3^2} + \frac{1}{3^3} + \frac{2}{3^4} + \frac{1}{3^5} + \frac{2}{3^6} + \ldots \infty$. (10 +10)

21. (a) Test the convergence of the series $\frac{x}{1} + \frac{1}{2}.\frac{x^2}{3} + \frac{1.3}{2.4}\frac{x^3}{5} + \frac{1.3.5}{2.4.6}\frac{x^4}{7} + \ldots \infty$.

(b) Sum to infinity the series $\left(1+\frac{1}{2}\right) + \left(\frac{1}{3}+\frac{1}{4}\right).\frac{1}{9} + \left(\frac{1}{5}+\frac{1}{6}\right)\frac{1}{9^2} + \ldots . \infty$. (10 + 10)

22. (a) Sum to infinity the series $1 + \frac{1+2}{2!} + + \frac{1+2+2^2}{3!} + \ldots . \infty$.

(b) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. Also find the equations of the shortest distance . (10 + 10)

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